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GCE

Edexcel GCE

Mathematics

Mechanics 5 M5 (6681)

June 2008

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Mark Scheme

Edexcel GCE

Mathematics

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathbf{d} = (7\mathbf{i} - 14\mathbf{j}) - (\mathbf{i} - 6\mathbf{j}) = (6\mathbf{i} - 8\mathbf{j})$	B1
	$(6k\mathbf{i} + k\mathbf{j}).(6\mathbf{i} - 8\mathbf{j}) = \frac{1}{2} \text{x} \frac{1}{2} \text{x} (2\sqrt{7})^2$	M1 A2 ft
	$28k = 7 \Rightarrow k = \frac{1}{4}$	D M1
	$\Rightarrow \mathbf{P} = \frac{3}{2}\mathbf{i} + \frac{1}{4}\mathbf{j}$	A1 **6**
2.	Aux Equn: $m^2 + 4m = 0 \Rightarrow m = 0$ or -4	M1
	$\mathbf{r} = \mathbf{A} + \mathbf{B}e^{-4t}$	A1
	$t = 0, \mathbf{r} = \mathbf{i} - \mathbf{j}$: $\quad \mathbf{A} + \mathbf{B} = \mathbf{i} - \mathbf{j}$	M1
	$\mathbf{v} = -4\mathbf{B}e^{-4t}$	M1
	$t = 0, v = -8\mathbf{i} + 4\mathbf{j}$: $\quad -4\mathbf{B} = -8\mathbf{i} + 4\mathbf{j}$	
	$\mathbf{B} = 2\mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = -\mathbf{i}$	A1 A1
	so, $\mathbf{r} = -\mathbf{i} + (2\mathbf{i} - \mathbf{j})e^{-4t}$	
	$= (2e^{-4t} - 1)\mathbf{i} - e^{-4t}\mathbf{j}$	A1 **7**
3.(a)	$\mathbf{R} = (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1
	$= (\mathbf{i} + \mathbf{k})$	A1 (2)
(b)	$\mathbf{G} + (5\mathbf{i} + \mathbf{j} - \mathbf{k}) \text{ x } (\mathbf{i} + \mathbf{k}) = (\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ x } (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + (4\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \text{ x } (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1 A2 ft
	$\mathbf{G} + (\mathbf{i} - 6\mathbf{j} - \mathbf{k}) = (-\mathbf{j} - \mathbf{k}) + (-4\mathbf{i} - 14\mathbf{j} - \mathbf{k})$	A3 ft
	$\mathbf{G} = (-5\mathbf{i} - 9\mathbf{j} - \mathbf{k})$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-5^2 + (-9)^2 + (-1)^2} = \sqrt{107}$ Nm	M1 A1 (9)
		11

4. (a)

$$-mg\,\delta t = (m + \delta m)(v + \delta v) + \delta m(U - v) - mv$$

$$-mg\,\delta t = mv + m\,\delta v + v\,\delta m + U\,\delta m - v\,\delta m - mv$$

$$-mg = m\frac{\mathrm{d}v}{\mathrm{d}t} + U\frac{\mathrm{d}m}{\mathrm{d}t}$$

$$m = M_0(1 - \tfrac{1}{2}t) \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t} = -\tfrac{1}{2}M_0$$

$$-M_0 g(1 - \tfrac{1}{2}t) = M_0(1 - \tfrac{1}{2}t)\frac{\mathrm{d}v}{\mathrm{d}t} - \tfrac{1}{2}M_0 U$$

$$U - g(2 - t) = (2 - t)\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{U}{(2 - t)} - 9.8 = \frac{\mathrm{d}v}{\mathrm{d}t} \quad *$$

M1 A2

A1

B1

M1

A1 (7)

(b)

$\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ when $t = 0 \Rightarrow \frac{U}{2} - 9.8 > 0$

$\Rightarrow \quad U > 19.6\,*$

M1

A1 (2)

(c)

$$v = \int \frac{U}{(2-t)} - 9.8\ \mathrm{d}t$$

$$= -U\ln(2-t) - 9.8t + C$$

M1

A1

$$t = 0, v = 0:\ 0 = -U\ln 2 + C \Rightarrow C = U\ln 2$$

$$\text{so, } v = U\ln\frac{2}{(2-t)} - 9.8t$$

M1

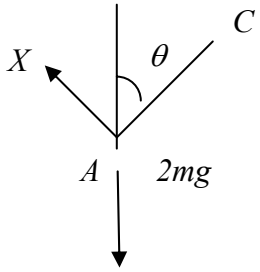
$$t = 1:\ v = U\ln 2 - 9.8$$

M1 A1 (5)

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Question Number	Scheme	Marks
5.(a)	$I = \frac{1}{3}m(9a)^2 + \frac{1}{2}2ma^2 + 2m(9a)^2$ $= 27ma^2 + ma^2 + 162ma^2$ $= 190ma^2$	M1 A1 A1 A1* (4)
(b)	M(L), $mg\frac{9a}{2}\sin\theta + mg9a\sin\theta = -190ma^2\ddot{\theta}$ $\ddot{\theta} = -\frac{9g}{76a}\sin\theta$ For small θ, $\sin\theta \approx \theta$, $\Rightarrow \ddot{\theta} = -\frac{9g}{76a}\theta$ so S.H.M. Period $= 2\pi\sqrt{\frac{76a}{9g}} = \frac{4\pi}{3}\sqrt{\frac{19a}{g}}$	M1 A2 M1 A1 DM1 A1 (7) **11**
6.	$\delta m = \pi a^2 \delta x . \frac{M}{\pi a^2 h} = \frac{M\delta x}{h}$ $\delta I = \frac{1}{4}\delta m.a^2 + \delta m.x^2$ $= \frac{M}{4h}(a^2 + 4x^2)\delta x$ $I = \int_0^h \frac{M}{4h}(a^2 + 4x^2)\mathrm{d}x$ $= \frac{M}{4h}\left[a^2 x + \frac{4}{3}x^3\right]_0^h$ $= \frac{M}{4}(a^2 + \frac{4}{3}h^2)$ $= \frac{M}{12}(3a^2 + 4h^2)$	M1 A1 M1 A1 M1 A1 M1 A1 M1 A1 **10**

7.(a)



$$\frac{1}{2}24ma^2\dot{\theta}^2 = 2mg.3a(1-\cos\theta)$$

$$2a\dot{\theta}^2 = g(1-\cos\theta)$$

M1 A1 A1

A1 (4)

(b)

$(\searrow)$

$$2mg\sin\theta - X = 2m3a\cos\theta$$

$$\text{M}(L),\quad 2mg.3a\sin\theta = 24ma^2\ddot{\theta}$$

$$\Rightarrow X = 2mg\sin\theta - 6ma\left(\frac{g\sin\theta}{4a}\right)$$

$$= \frac{mg\sin\theta}{2} *$$

M1 A2
M1 A1

DM1

A1 (7)

(c)

$$\theta = \pi : 2a\dot{\theta}^2 = g(1-\cos\pi)$$

$$\dot{\theta} = \sqrt{\frac{g}{a}}$$

$$6a.I = 24ma^2\sqrt{\frac{g}{a}}$$

$$\Rightarrow I = 4m\sqrt{ag}$$

M1

A1

M1 A1

A1

(5)

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